Fast analysis of massively parallel systems

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(supervised by Jeremy Bradley, joint work with Richard Hayden)

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How to make coffee?

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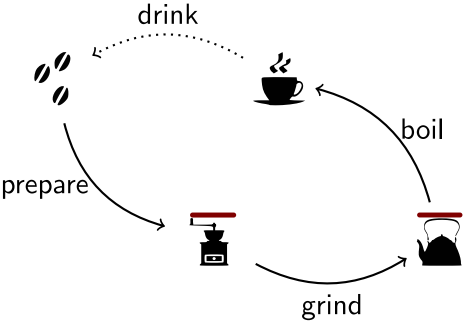
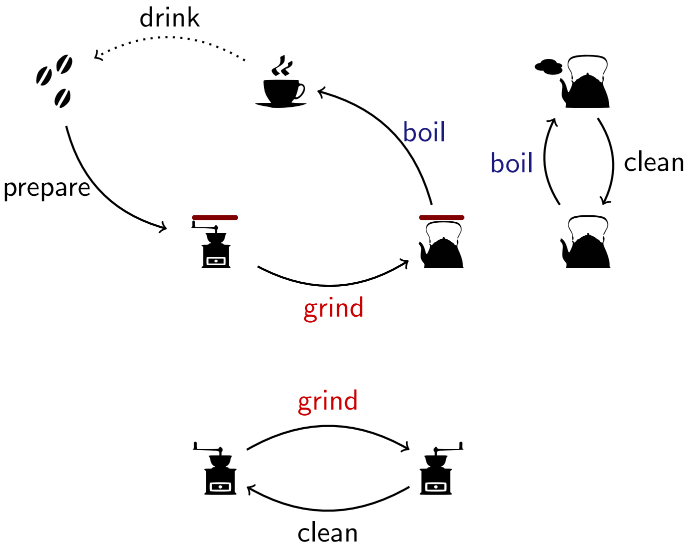


How do 100 000 people make coffee?

Limited number of grinders and kettles



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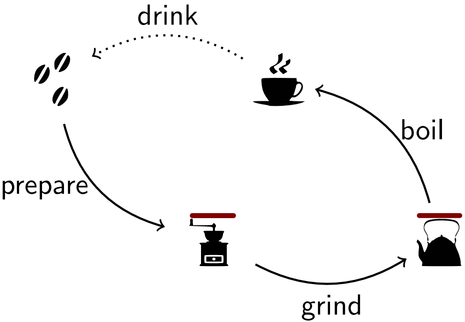
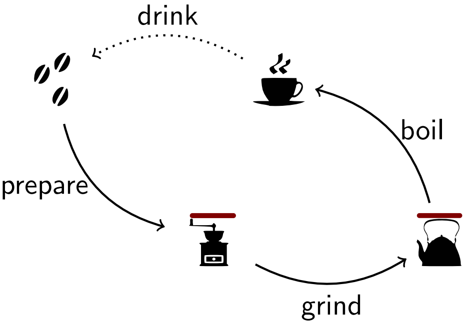


$\times$100 000 copies

How do 100 000 people make coffee?



$\times$100 000 copies

Gives $4^{100\,000} \approx 9.98 \times 10^{60205}$ states

If counting each of , , , 1.667×10^{14} states. Still too many.

ODE approximation

Counts $\text{beans}(t)$, $\overline{\text{grinder}}(t)$, $\overline{\text{kettle}}(t)$, $\text{coffee}(t)$ represent the system

Can get ODEs for the means $\mathbb{E}[\text{beans}(t)]$, $\mathbb{E}[\overline{\text{grinder}}(t)]$, $\mathbb{E}[\overline{\text{kettle}}(t)]$, $\mathbb{E}[\text{coffee}(t)]$:

ODE approximation

Counts (t), (t), (t), (t) represent the system
Can get ODEs for the means $\mathbb{E}[(t)]$, $\mathbb{E}[(t)]$, $\mathbb{E}[(t)]$, $\mathbb{E}[(t)]$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbb{E}[(t)] = r_{\text{prepare}}\mathbb{E}[(t)] - r_{\text{grind}}\mathbb{E}[\min((t), (t))]$$

$$\approx r_{\text{prepare}}\mathbb{E}[(t)] - r_{\text{grind}}\min(\mathbb{E}[(t)], \mathbb{E}[(t)])$$

ODE approximation

Counts $\text{beans}(t)$, $\overline{\text{grinder}}(t)$, $\overline{\text{kettle}}(t)$, $\text{cup}(t)$ represent the system
Can get ODEs for the means $\mathbb{E}[\text{beans}(t)]$, $\mathbb{E}[\overline{\text{grinder}}(t)]$, $\mathbb{E}[\overline{\text{kettle}}(t)]$, $\mathbb{E}[\text{cup}(t)]$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbb{E}[\overline{\text{grinder}}(t)] = r_{\text{prepare}}\mathbb{E}[\text{beans}(t)] - r_{\text{grind}}\mathbb{E}[\min(\overline{\text{grinder}}(t), \text{grinder}(t))]$$

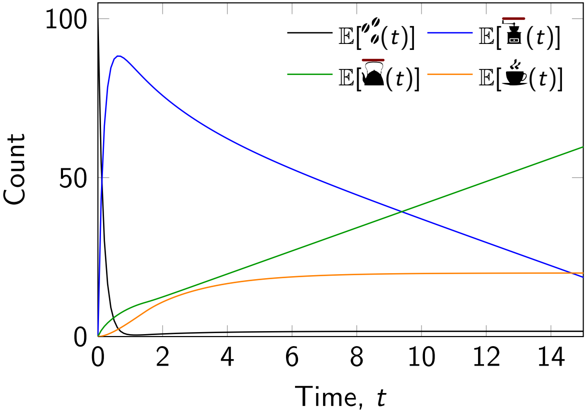
$$\approx r_{\text{prepare}}\mathbb{E}[\text{beans}(t)] - r_{\text{grind}}\min(\mathbb{E}[\overline{\text{grinder}}(t)], \mathbb{E}[\text{grinder}(t)])$$

Similarly for higher moments like

$$\mathbb{E}[\overline{\text{grinder}}(t)^2], \mathbb{E}[\text{grinder}(t)\,\text{kettle}(t)^3], \mathrm{Var}[\text{beans}(t)], \ldots$$

Example

(coffee beans)(0) = 100, (grinder)(0) = 5, (kettle)(0) = 15:



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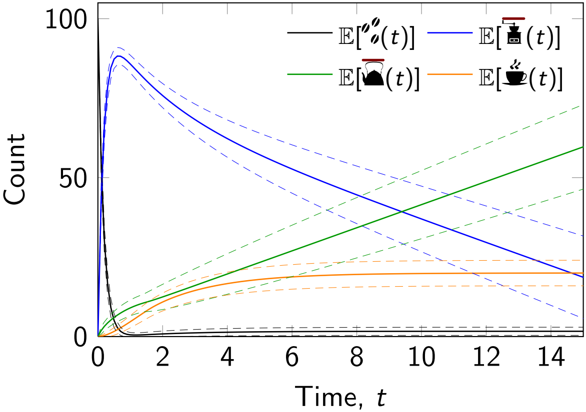
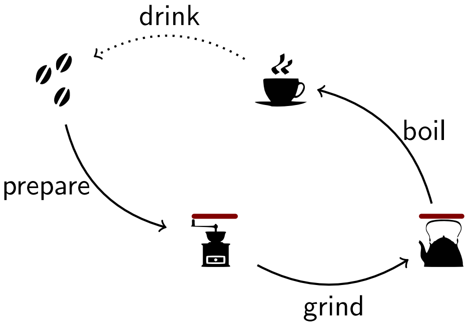


Passage times



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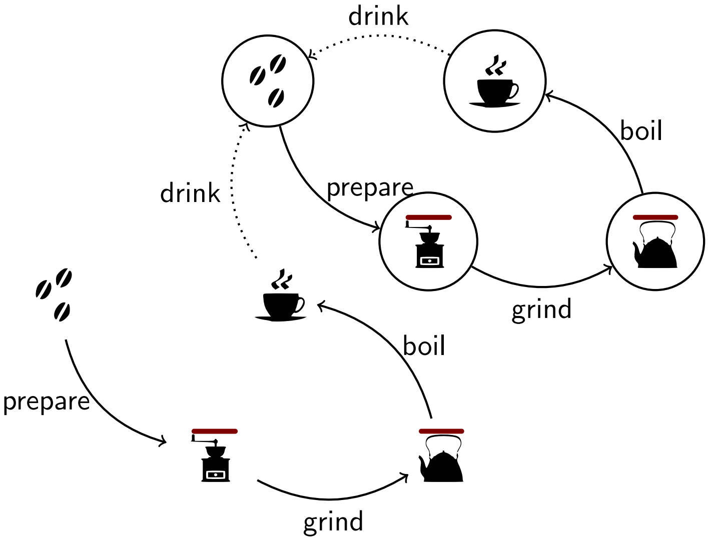
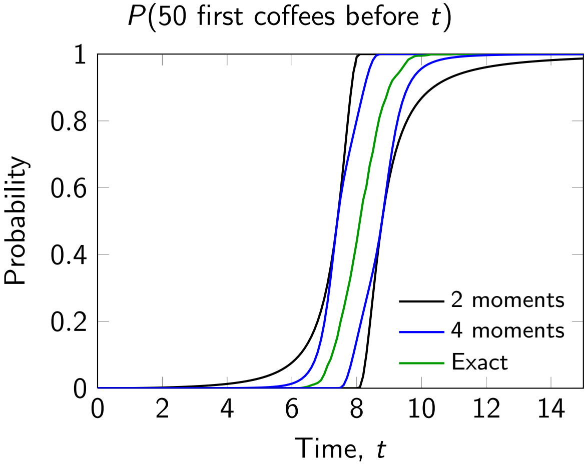


Passage times

Bounds on passage time probabilities from higher moments:



Accumulated quantities

Can derive ODEs for:

$$\frac{\mathsf{d}}{\mathsf{d}t}\mathbb{E}\left[\int_0^t \text{⁖}(u)\mathsf{d}u\right] = \mathbb{E}[\text{⁖}(t)]$$

Accumulated quantities

Can derive ODEs for:

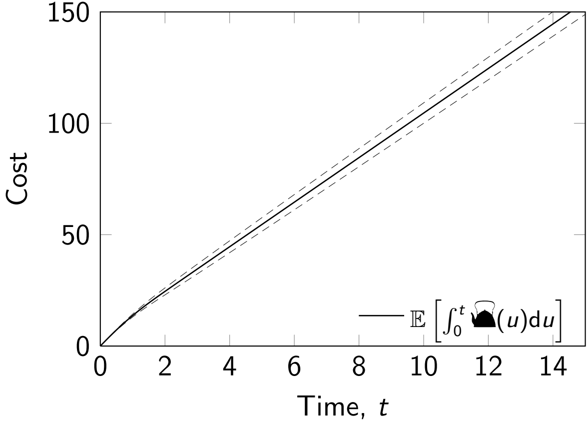
$$\frac{\mathsf{d}}{\mathsf{d}t}\mathbb{E}\left[\int_0^t \text{〃}(u)\mathsf{d}u\right] = \mathbb{E}[\text{〃}(t)]$$

But also for higher moments

$$\mathbb{E}\left[\left(\int_0^t \text{〃}(u)\mathsf{d}u\right)^2\right] \qquad \mathbb{E}\left[\text{〃}(t)\int_0^t \text{〃}(u)\mathsf{d}u\right]$$

Accumulated quantities

Cost of using the kettles:



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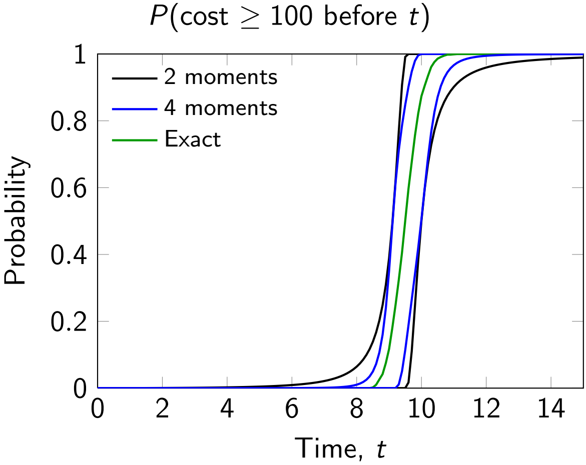
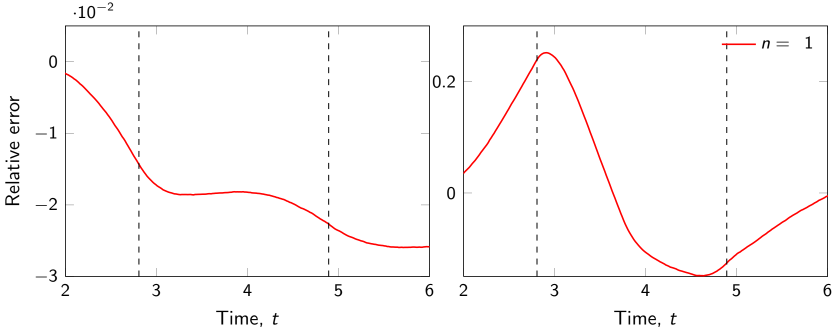


Approximations

Take (coffee beans)$(0) = n \times 100$, (grinder)$(0) = n \times 5$, (kettle)$(0) = n \times 15$ $n = 1, 4, 16, 64$



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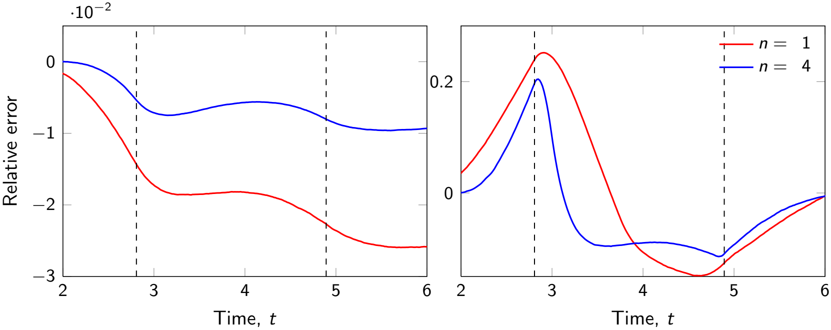

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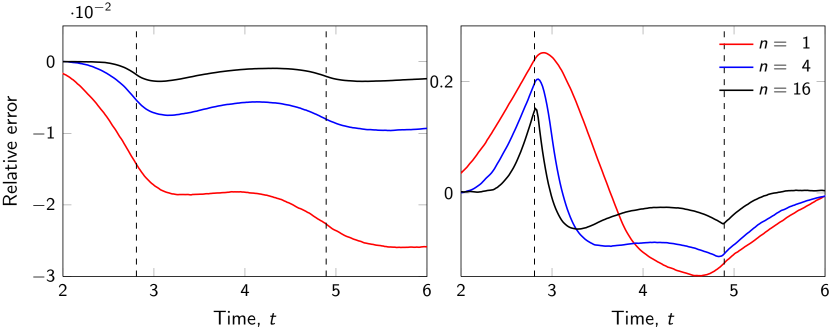

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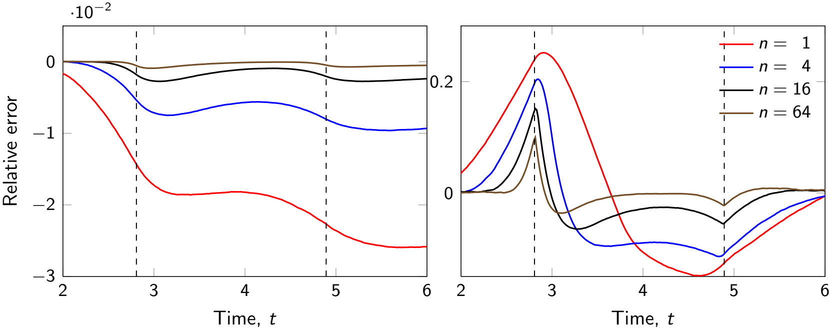

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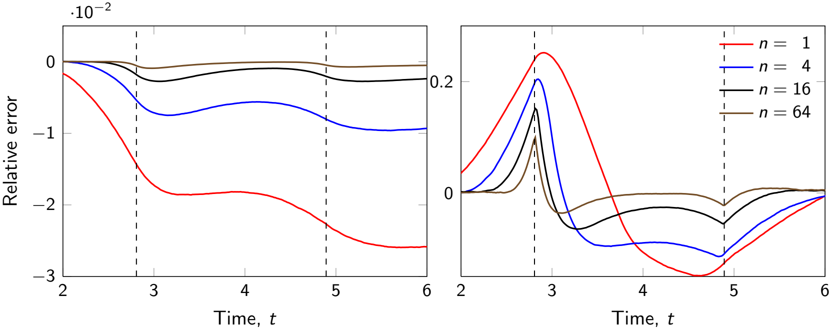

- Kurtz
- Switch points

Summary & Further work

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- ODEs approximating various quantities in large models
- Faster than simulations
- Implementation for models described in PEPA stochastic process algebra

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Future work:

- Different ways of synchronization
- More general distributions (phase type)
- Optimizing parameters
- Real applications

Thank you!